

OPTIMIZATION OF THE LATTICE
FOR INTRABEAM SCATTERING
FOR
SHORT BUNCHES OPERATION MODE
(60° PHASE ADVANCE CELL)

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The Collider is made of 6 periods -

Each period is made of an arc with radius of curvature $R_0 = 381.2325 \text{ m}$ and a long straight section of length $L_s = 289.7358 \text{ m}$. In the arc we are going to install regular FODO cells each with length $2L$. The bending angle per half regular cell is

$$\theta = 2\pi/N \quad (1)$$

where

$$N = 2\pi R_0 / L \quad (2)$$

is the number of $\frac{1}{2}$ regular cells.

We require the phase advance per cell is 60° . For a thin lens approximation this gives : for the quadrupole gradient B'

$$\frac{B' l_Q}{B_f} L = 1 \quad (3)$$

l_Q , quadrupole length

B_f , beam magnetic rigidity

$$\begin{aligned}\beta_{\max} &= 2\sqrt{3} L \\ \beta_{\min} &= 2/\sqrt{3} L\end{aligned}\tag{4}$$

$$\begin{aligned}\eta_{\max} &= 5L\theta \\ \eta_{\min} &= 3L\theta\end{aligned}\tag{5}$$

Let us define the averages

$$\bar{\beta} = (\beta_{\max} + \beta_{\min})/2 = 2.3094 L \tag{6}$$

$$\bar{\eta} = (\eta_{\max} + \eta_{\min})/2 = 4 L\theta \tag{7}$$

Taking into account (1) and (2), (6) becomes

$$\bar{\eta} = 4 \frac{L^2}{R_0} \tag{8}$$

Therefore if the cell half-length L is given, we obtain $\bar{\beta}$ and $\bar{\eta}$ from (6) and (8) -

This is shown in the Table at the end of the note -

We have done intrabeam scattering calculation at the computer for a smooth machine defined by the parameters $\bar{\beta}$ and $\bar{\eta}$. We have calculated the diffusion rate in energy τ_E^{-1} and betatron size τ_β^{-1} for a 0.001 Amp-particle bunched beam at 100 GeV/A for Gold ($A = 197$, $Z = 79$). We have assumed a normalized emittance

$$\epsilon_N = 4\pi \text{ mm-mrad}$$

and an rms energy spread within the bunch

$$\sigma_E/E = 4 \times 10^{-4}$$

The diffusion rates are shown in the Table. The energy diffusion rate decreases whereas the betatron diffusion rate increases with the half-length L of a cell.

These diffusion rates affect the luminosity lifetime with the same order.

For a luminosity of $\approx 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$ one requires 57 short bunches each with $N = 6.24 \times 10^8$ and an rms bunch length of $\sigma_t = 10 \text{ cm}$. This corresponds to a peak current of 0.12 Amp-particle.

In the Table we give the luminosity

diffusion time

$$t_L = \frac{1}{120 (\tau_\alpha^{-1} + \tau_\beta^{-1})}$$

As one can see the shorter the length cell the longer the luminosity lifetime.

Possibly the luminosity lifetime could be actually longer than the values reported by $\sim 50\%$ because of the long straight sections where the dispersion is zero and there is no local beta from diffusion due to intrabeam scattering.

Therefore a cell length of about $2L = 22\text{ m}$ seems to be adequate for a luminosity lifetime of about one hour which is a long period of time compared to the filling time of 2×1 minute.

In the same Table we report the approximate estimate of the transition energy γ_T

$$\gamma_T \approx \sqrt{R/\bar{m}}$$

where $R = 610.17\text{ m}$ is the average radius.

Our choice, marked with a star, corresponds to $\gamma_T \approx 22$.

We also give the maximum coupling impedance Z/n allowed for ~~short~~ longitudinal beam stability according to the formula

$$|Z/n| \lesssim \frac{E|\eta|}{e I_p} \left(2 \frac{\alpha_E}{E}\right)^2 \frac{A}{Z^2}$$

The beam we have specified above corresponding to a short bunch mode of operation has an invariant longitudinal emittance of

$$S = 0.25 \text{ eV/A-sec / bunch}$$

for 95% of the particle distribution -

Can one produce such a bunch from the AGS?

With our choice of $L \approx 11 \text{ m}$ we have

$$\beta_{\max} = 37.5 \text{ m}$$

$$\eta_{\max} = 1.54 \text{ m}$$

Let us consider the case of Gold at 5 GeV/A with a small momentum spread

$$\Delta p/p = 2 \times 10^{-3}$$

and

$$\epsilon_{H,V} = 0.8 \pi \text{ mm-mrad}$$

The maximum beam full height is

$$a_v = 2 \times \sqrt{0.8 \times 37.5} \text{ mm} = 11 \text{ mm}$$

and the maximum full width is

$$a_H = 2 \sqrt{(1.54 \times 1)^2 + (0.8 \times 37.5)} = 11.4 \text{ m}$$

or at most

$$a_H = \left[(1.54 \times 2) + 2 \sqrt{0.8 \times 37.5} \right] \text{ mm} = 14 \text{ mm}$$

Sketch of a Regular Cell (Approximated)

Take $L = 10.8 \text{ m}$

Quadrupole length $l_q = 1.2 \text{ m}$

BP for Gold at 100 GeV/A = 800 T-m

Quadrupole Gradient = 62 T/m

Bore Radius = 4 cm

Field at Pole Tip = 2.5 T

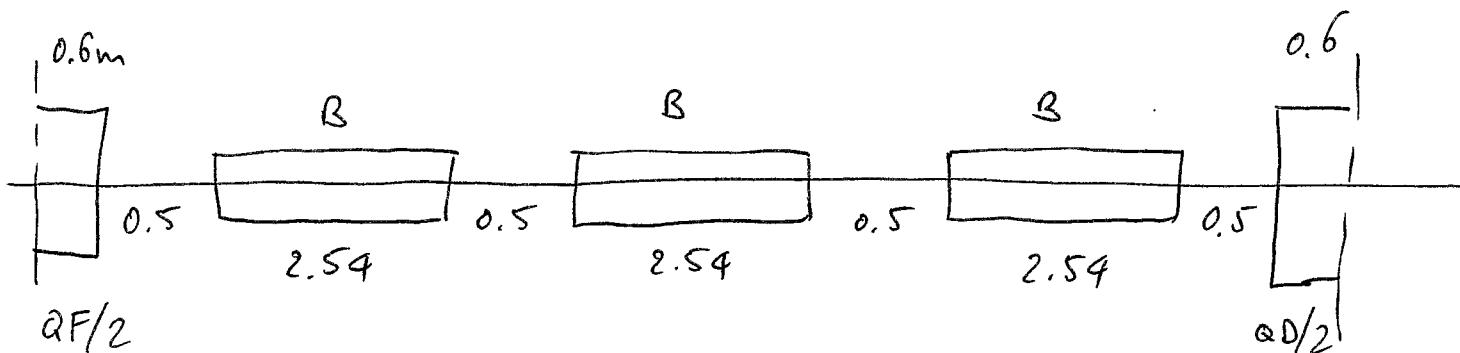
$N = 220$ half-cells

$\theta = 28.56 \text{ mrad}$ ~~bending angle / half-cell~~

Assume 3 bending magnet / half cell

Take $B = 3.0 \text{ T}$ then $3l_B = 7.616 \text{ m}$

That is $l_B = 2.54 \text{ m}$



I believe one single curved dipole is better

| L | $\bar{\beta}$ | $\bar{\eta}$ | τ_E^{-1} | τ_B^{-1} | t_L | δ_T | Z/n |
|-------|---------------|--------------|---------------|---------------|-----------|------------|-------|
| m | m | m | h^{-1} | h^{-1} | hours | | ohm |
| 2.16 | 5 | 0.05 | - | - | - | 110 | 0.3 |
| 4.33 | 10 | 0.2 | - | - | - | 55 | 3.9 |
| 6.49 | 15 | 0.44 | .0044 | .0034 | 1.1 | 37 | 10.6 |
| 8.66 | 20 | 0.786 | .0033 | .002 / .0073 | 1.6 / 0.8 | 28 | 19.8 |
| 10.82 | 25 | 1.23 | .0025 | .0051 / .01 | 1.1 / 0.7 | 22 | 33 * |
| 12.99 | 30 | 1.77 | .002 | .0076 / .0119 | 0.9 / 0.6 | 18.6 | 47 |
| 15.15 | 35 | 2.41 | .0016 | .0133 | 0.56 | 16 | 64 |
| 17.32 | 40 | 3.146 | .0013 | .0143 | 0.53 | 14 | 84 |
| 19.48 | 45 | 3.98 | .0011 | .0181 | 0.43 | 12.4 | 108 |
| 21.64 | 50 | 4.92 | .0009 | .0211 | 0.38 | 11 | 137 |